International Journal of Thermophysics, Vol. 27, No. 1, January 2006 (© 2006)
DOI: 10.1007/s10765-006-0016-7

Constructal Enhancement of Heat Conduction with Phase Change[1]

Ai-Hua Wang,[2] **Xin-Gang Liang,**[2,3] **and Jian-Xun Ren**[2]

For enhancing the heat energy release from a phase change material (PCM) the conductive fin is designed according to a proposed constructal rule, in which the high conductivity material (HCM) should be located at the place where the heat flux density is the largest. The local temperature gradient integration over time is taken as a criterion to determine where to distribute the limited HCM during a given time period. This rule is applicable to heat conduction of steady and unsteady conditions, as well as to the problems with and without phase change. Numerical simulations show that the constructal design of a conductive fin has much better performance than arbitrary ones. The constructal rule is an effective technique that designs the fin with high performance for enhancing heat conduction.

KEY WORDS: constructal optimization; finite element method; heat conduction; phase change.

1. INTRODUCTION

A spacecraft will have a significant fluctuation of heat production due to an increasing number of periodically working power devices. For instance, the constant heat production of the instrument cabinet on the China Brazil Earth Resource Satellite (CBERS-1) is only 73 W while the peak pulse power is 526 W [1]. It is an attractive idea to adopt the phase change material (PCM) to prevent the temperature of high power electronics from going too high [2–6]. The PCM melts to absorb heat energy and reduce

---

[1] Paper presented at the 7th Asian Thermophysical Properties Conference, August 23–28, 2004, Hefei and Huangshan, Anhui, P. R. China.

[2] Department of Engineering Mechanics, Tsinghua University, Education Ministry Key Lab of Enhanced Heat Transfer and Energy Conservation, Beijing 100084, P. R. China.

[3] To whom correspondence should be addressed. E-mail: liangxg@tsinghua.edu.cn

the temperature of the high pulse power electronics. When the pulse electronics is off, the PCM solidifies gradually to restore cold energy. The melting process of a PCM is a pure heat conduction process in microgravity and in some cases on the ground.

It is necessary to enhance the melting process due to the characteristics of the very low thermal conductivity of a PCM. There are two optimization targets. The first is that the time needed to melt all of the PCM should be the shortest for a conducting material with a given volume fraction. The second is that the volume fraction of the conducting materials needed should be the least one for melting a certain fraction of the PCM. Wang [7] studied the inner finned casing-tube in order to expedite the solidification process of a PCM. Zhang and Liang [8] studied the effective thermal conductivity with a high conductivity material (HCM) powder in low conductivity materials. Fukai et al. [9] used carbon fibers to enhance the effective thermal conductivity of an energy storage medium. What they studied are all direct or normal problems, that is, studies of heat transfer characteristics with a given arrangement of HCMs. The inverse problem, that is, the optimization of the arrangement of the HCM, is more challenging. Bejan [10] developed a constructal law to optimize the heat rejection problem and invoked global pursuit of performance subject to global constraints in a freely morphing configuration. A typical example is the volume-to-point problem. The problem is to effectively conduct the heat uniformly generated in a finite-size volume to a small patch (point), which is located on its boundary, by constructing a high conductivity path. After that, this method was extended to unsteady conduction [11]. A smallest volume element system was first considered and the conducting path is deduced that can cool the peak temperature in minimum time. Then they developed the constructal rule for a larger system by covering it with a number of element systems. Zamfirescu and Bejan [12] further applied this method to two-phase flow for cooling a surface. The system is a surface with uniform heating per unit area, which is cooled by a network with evaporating two-phase flow. Illustrations are based on the design of the cooling network for a skating rink. The influence of the phase change of coolant in the pipe is treated as a boundary with a given heat transfer coefficient. This constrained minimization of global resistance was further extended to river drainage basins [13]. Guo et al. [14] and Cheng et al. [15–17] developed a constructal optimization method in which a uniform temperature gradient and the least dissipation principle of heat transport potential capacity were proposed to study the optimum arrangement of a HCM in the low conductivity matrix for steady-state conduction problems. No reports are found on the optimum distribution of HCMs for transient conduction in PCMs.

The goal of this work is to extend further the constructal optimization to transient cases with phase change and try to develop a general rule that can optimize the arrangement of conductive materials. The constructal optimization not only provides the basic idea and specific method but also involves a fundamental sphere of heat transfer theory, that is, to obtain the optimum heat transfer effect through optimizing the local behavior of the heat transfer process.

2. CONSTRUCTAL OPTIMIZATION OF HEAT CONDUCTION WITH PHASE CHANGE

Constructal optimization of heat conduction with phase change mimics the growth of plants in nature. The HCM is treated as the plant root, and the matrix (PCM) acts as the soil. The condition of the matrix, such as the latent heat, thermal conductivity, and heat capacity, and the boundary, influence the growth of the "plant". Following the natural selection, the HCM gradually grows into the heat conduction network to acclimatize itself to the environment. Finally, the optimum and highly effective heat transport path is constructed to absorb and transport the "nutrition", the apparent and latent heat, from the PCM.

The evolutionary process for the constructal optimization method includes a generation aspect and a degeneration aspect. For steady-state heat conduction, the generation is to improve the heat transport ability at the place where the local temperature gradient is the largest in the domain while the degeneration is to withdraw HCM at the place that has the lowest local temperature gradient. A high temperature gradient corresponds to a high heat transport potential. We can most effectively increase the local heat flux if we locate HCM at this place. As a result, the local temperature gradient will be decreased. At the location of the low temperature gradient, the heat transfer potential is lower than that at the location of the high temperature gradient. Thus, we can remove HCM from the location of low temperature gradient to the location of high temperature gradient if the amount of HCM is limited. The generation and degeneration will make the temperature gradient in the whole heat conduction domain more and more uniform.

The constructal optimization of heat conduction with phase change is different. The heat conduction with phase change is an unsteady process. The temperature gradient at any location in the physical domain varies continuously with time. The locations of the maximum and minimum temperature gradients in the domain also change with time. The arrangement position of the HCM cannot be determined simply according to the temperature gradient.

The product of thermal conductivity and temperature gradient integration over time ($\int_0^t |\nabla T|\, dt$) in the whole melting time interval of the PCM is taken as the criterion to determine the arrangement position of the HCM for the unsteady case. Its physical meaning is the quantity of total heat transported during the melting time interval. The generation rule of constructal optimization of heat conduction with phase change is to grow the HCM at the position with the maximum total heat transported during the time interval. At this position the averaged temperature gradient over time is the largest before or during the process of generation. The degeneration rule is to withdraw the HCM at the position with minimum total heat transported during the time interval. The averaged temperature gradient over time is the least before or during the process of degeneration. The rules are based on the accumulated results, not on the instant temperature profile. The basic idea is to apply the limited HCM to the position where it can raise the heat conduction most during the given time period, and as a result, the time-averaged temperature gradient becomes uniform. The process of generation and degeneration of the HCM can be completed by numerical simulation. The purpose is to make the time interval needed for the melting of PCM the shortest for the condition with a given volume fraction of the HCM.

3. MATHEMATICAL MODEL AND NUMERICAL SIMULATION

For the melting problem of a PCM the enthalpy model of Shamsundar and Sparrow [18], with the enthalpy as an unknown function, is used. The governing equation can be expressed as follows:

$$\rho \frac{\partial h}{\partial t} = k \nabla^2 T \tag{1}$$

$$(\rho, k) = \begin{cases} k_{\mathrm{s}}, \rho_{\mathrm{s}} & \text{for } h < h_{\mathrm{s}}^* \\ k_{\mathrm{l}}, \rho_{\mathrm{l}} & \text{for } h > h_{\mathrm{l}}^* \end{cases} \tag{2}$$

$$T - T_{\mathrm{f}} = \begin{cases} (h - h_{\mathrm{s}}^*)/c_{\mathrm{s}} & \text{for } h < h_{\mathrm{s}}^* \\ 0 & \text{for } h_{\mathrm{s}}^* \leqslant h \leqslant h_{\mathrm{l}}^* \\ (h - h_{\mathrm{l}}^*)/c_{\mathrm{l}} & \text{for } h_{\mathrm{l}}^* < h \end{cases} \tag{3}$$

where h is the enthalpy, ρ is the density, k is the thermal conductivity, T is the temperature, c is the specific heat; the superscript * denotes the saturation parameter at the phase change temperature T_{f}, and the subscripts s and l denote the solid and liquid phases. The relation between enthalpy and temperature is

$$h = \int_{T_f}^{T} \rho_s c_s \, dT, \quad \text{for } T \leqslant T_f$$

$$h = \int_{T_f}^{T} \rho_l c_l \, dT + \rho_l \gamma, \quad \text{for } T > T_f$$

where γ is the latent heat.

Because the energy conservation equation is established with enthalpy as the unknown variable, and not the temperature, the aforementioned governing equation is suitable for the entire domain including the solid and liquid phases and their interface. The advantage is that it is not necessary to solve the energy equation of the phase change interface and to determine the interface position through it. When phase change takes place, the enthalpy will change to that of the corresponding phase. The position of the interface can be determined according to the value of the enthalpy. The numerical calculation procedure is simple and convenient. As a demonstration only, a uniform initial condition is considered. It can be realized through the finite element method (FEM), and the element is a square with four nodes. The main steps are as follows: (a) grid mesh generation; (b) computation of the temporal temperature field; (c) calculation of the temperature gradient field according to the temperature field; (d) integration of the temperature gradient at every node within the domain in the given time interval; (e) determination of the volume element to be generated or degenerated according to the integration of the temperature gradient and the evolutionary principle; and (f) repetition from the second to the fifth steps until the given volume of the HCM is depleted.

4. RESULTS AND DISCUSSION

4.1. Effect of Constructal Optimization

The numerical simulation of the special heat conduction example is made to justify that the system has the best heat transfer performance when the thermal conductivity is distributed according to the constructal optimization. Figure 1 shows the structure of the computation domain with phase change. It is a square-shaped PCM. The lateral length is 50 mm, and a side of a square element is 1 mm long. The cold energy

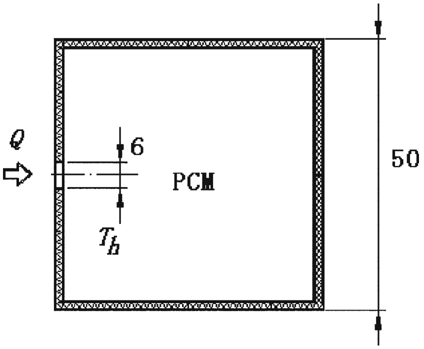

Fig. 1. Heat conduction domain with phase change.

stored in the PCM will be released from a 6 mm hatch at the left side at a temperature $T_h = 60°\text{C}$, and the other boundary is adiabatic. The parameters of the PCM are taken as those of ice, and the parameters of the HCM are those of aluminum. The initial temperature in the whole domain is $-1°\text{C}$. The purpose is to design the distribution of the given HCM in the domain to ensure that the time needed for melting the PCM is the shortest. If the melting times before and after optimization are compared, the enhancement effect of the distribution of the HCM is shown very clearly.

The distribution of thermal conductivity is realized by insertion and removal of the HCM in the domains that are determined by the generation or degeneration discussed above. Figures 2 and 3 are two man-made structures of the HCM. The result of optimum distribution of the HCM is shown in Fig. 4. The volume fraction of the HCM is 10% for the three cases. The melting time and cold release power are compared in

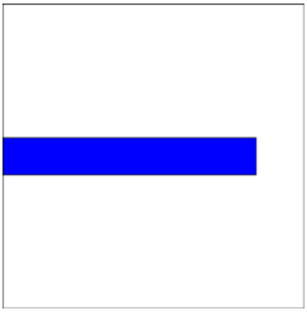

Fig. 2. First conducting structure in phase change material.

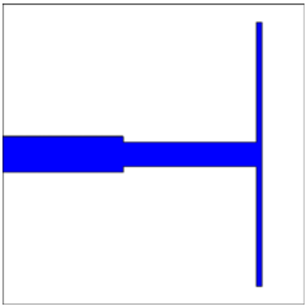

Fig. 3. Second conducting structure in phase change material.

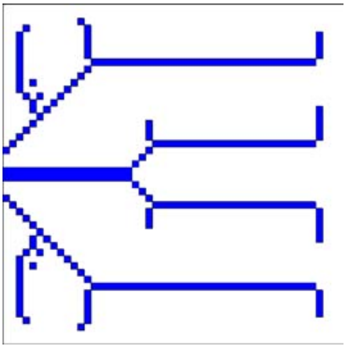

Fig. 4. Conducting structure after optimization with thermal conductivity ratio of 118.0 between high conductivity material and phase change material.

Table I. Comparisons of Melting Time and Cold Release Power of Three Arrangement Schemes

	First Structure	Second Structure	Constructal Optimization
Melting time of ice (s)	3142	2541	923
Cold release power (W)	258	255	814

Table I. The melting time needed for the first structure is 3142 s and the time needed for the second is 2541 s. The melting time needed for the constructal structure is 923 s. There are significant reductions in the melting time of the constructal structure. Consequently, the average cold release power is increased as well, as shown in Table I.

4.2. Effect of Thermal Conductivity Ratio on Constructal Optimization

The conductivity ratio has an influence on the final constructal structures. Comparisons of Figs. 4–7 demonstrate this effect. The latent heat is $334\,\mathrm{kJ}\cdot\mathrm{kg}^{-1}$, and the integration time is limited to 1755 s for these figures.

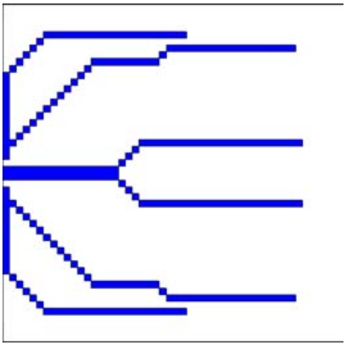

Fig. 5. Conducting structure after optimization with thermal conductivity ratio of 53.5 between high conductivity material and phase change material.

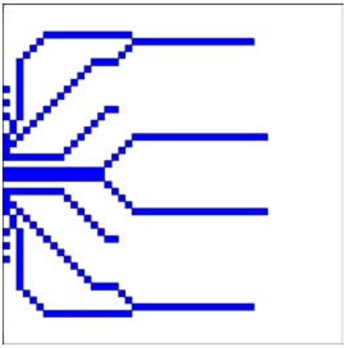

Fig. 6. Conducting structure after optimization with thermal conductivity ratio of 28.0 between high conductivity material and phase change material.

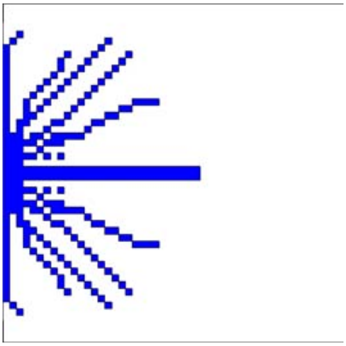

Fig. 7. Conducting structure after optimization with thermal conductivity ratio of 6.5 between high conductivity material and phase change material.

The thermal conductivity ratios of HCM to PCM are 118.0, 53.5, 28, and 6.5, respectively.

The constructal structure with phase change is different from that for steady-state heat conduction problem [14, 19]. The structure is more compact, and the branches of the HCM are closer to each other in the latter case. But in the case with phase change, the HCM is distributed much more uniformly and the "root" of the HCM is always fasciculated. The configuration of the HCM is stretched out enough and an appropriate branch distance is maintained because the "nutrition", the cold energy, is very rich and widespread. Every branch can be fully used to transport local cold energy. At a low conductivity ratio, the root of the HCM is substantially smaller and has more branches, and there are no divarications in each branch. As a result, long distance transport is not an advantage of low conductivity materials. These trends are very clear in Fig. 7 with a thermal conductivity ratio of 6.5. For a high conductivity ratio the root of the HCM is deep-rooted and the branches may have divarications. When the root reaches the boundary, it begins to grow breadthwise.

4.3. Effect of Latent Heat on Constructal Optimization

In Figs. 6, 8, and 9, the conductivity ratio is fixed at 28 and the integration time is 1755 s. The values of latent heat are 334.0, 167.0, and 83.5 $\mathrm{kJ \cdot kg^{-1}}$, respectively. Large latent heat leads to a root with more branches that are relatively lower. The main root begins to divaricate at a relatively low position. The nutrition is rich enough so that the root need not grow very deeply. When the latent heat is relatively small, the HCM has to root deeper to absorb more cold energy. The distance between each branch is relatively large. The main root begins to branch off at the relatively high position.

4.4. Effect of Integration Time on Constructal Optimization

Figures 4, 10, and 11 demonstrate the effect of integration time. They have the same conductivity ratio, 118.0 and latent heat, 334.0 kJ · kg^{-1}. The integration times are 1755, 200, and 20 s, respectively. When the integration time is short, the root of the HCM becomes substantially smaller and compact and congregates at the vicinity of the heat source. Only the

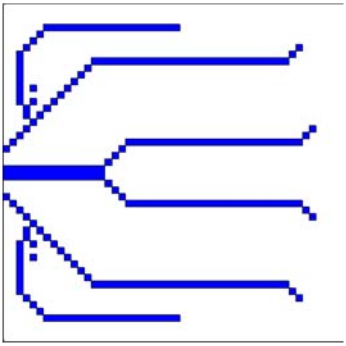

Fig. 8. Conducting structure after optimization with latent heat 167.0 kJ · kg^{-1}.

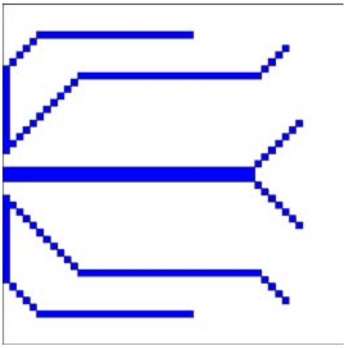

Fig. 9. Conducting structure after optimization with latent heat 83.5 kJ · kg^{-1}.

PCM nearby the heat source can melt in the given limited time. The envelope of the root tips looks like an arc. When the integration time becomes longer, the root of the HCM becomes more stretched and more uniformly distributed in the PCM. The shape variation of the root becomes less with longer time.

There are two choices that make full use of the latent heat of the PCM. One is to extend the time for melting the PCM, and another is to increase the volume fraction of the HCM. The best design is to melt all the PCM using the proper volume fraction of the HCM in a proper time.

From the above results, it is demonstrated that the conductivity ratio, latent heat, and melting duration all affect the structure of the conducting materials. Other factors, such as percentage of high conducting mate-

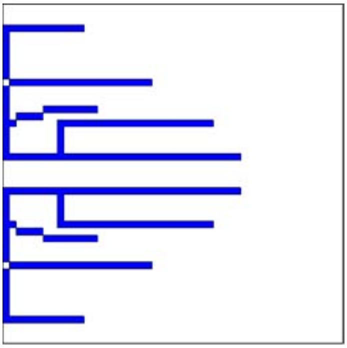

Fig. 10. Conducting structure after optimization with integration time 200 s.

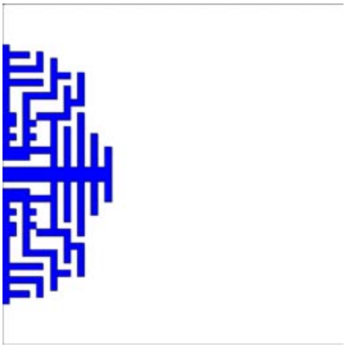

Fig. 11. Conducting structure after optimization with integration time 20 s.

rials, heating/cooling position and manner, could also have impacts on the structure. The optimized structures of high conducting materials in this work are quite similar to those in Ref. 13. This is because in both cases the requirements are the same: minimum resistances to heat flux and to liquid flow.

5. CONCLUSION

The idea of heat conduction constructal optimization can be extended from the steady-state problem to unsteady-state problem, from the heat conduction without phase change to the one with phase change if the local temperature gradient integration over time is used as the criterion for the arrangement of HCM. Examples of a man-made HCM structure are compared with the structure obtained by constructal optimization. The arrangement of HCM obtained by the constructal optimization exceeds the man-made structure in melting time and the average cold release power.

Similar to the steady case the conductivity ratio has influence on the structure of the HCM in heat conduction with phase change. A lower conductivity ratio leads to a substantially smaller root structure with more branches. But the root is not as compact as it is in the steady-state case, and it distributes more uniformly in the PCM.

The distinguishing feature for conduction with phase change is that both the phase change and time period have effects on the optimized structure of the HCM. Short-time intervals result in a substantially smaller root distribution of the HCM. As time increases, the root grows deeper and widespread. A large latent heat will also produce a substantially smaller root structure of the HCM because the latent heat is rich enough for the HCM root to transport in a given time. When the latent heat is small, the root become deep and its branches become sparse.

REFERENCES

1. Z. F. Pang, *Spacecraft Eng.* **11**:93 (2002) [in Chinese].
2. B. G. Schelden and J. O. Golden, *Proc. 7th AIAA Thermophysics Conf.*, San Antonio, Texas (1972) pp. 296–302.
3. J. P. Kirkpatrick and P. J. Brennan, *Proc. 8th AIAA Thermophysics Conf.*, (Palm Springs, California), (1973), pp. 715–717.
4. A. Haiji-Sheikh, J. Eftekhar and D. Y. S. Lou, *Proc. 3rd AIAA/ASME Joint Thermophysics, Fluid, Plasma & Heat Transfer Conf.* (St. Louis, Missouri, 1982), pp. 094–096.
5. D. Gilmore, *Satellite Thermal Control Handbook Handbook* (Aerospace Corp., California, 1984), pp. 138–139.

6. A. Abhat and M. Groll, *Proc. AIAA/ASME Thermophysics & Heat Transfer Conf.* (Boston, Massachusetts, 1974), pp. 138–139.
7. Y. M. Wang, *J. Chong Qing University* **17**:121 (1994) [in Chinese].
8. Y. P. Zhang and X. G. Liang, *Material Design* **16**:91 (1995).
9. J. Fukai, M. Kanou, Y. Kodama and O. Miyatake, *Energy Conversion Management* **41**:1543 (2000).
10. A. Bejan, *Int. J. Heat Mass Transfer* **46**:799 (1997).
11. N. Dan and A. Bejan, *J. Appl. Phys.* **84**:3042 (1998)
12. C. Zamfirescu and A. Bejan, *Int. J. Heat Mass Transfer* **46**:2785 (2003).
13. M. R. Errera and A. Bejan, *Fractals* **6**:245 (1998).
14. Z. Y. Guo, X. G. Cheng and Z. H. Xia, *Chinese Sci. Bull.* **48**:406 (2003).
15. X. G. Cheng, Z. X. Li and Z. Y. Guo, *Sci. China (Series E)* **46**:296 (2003).
16. X. G. Cheng, Z. H. Xia, Z. X. Li and Z. Y. Guo, *J. Eng. Thermophys*. **23**:715 (2002) [in Chinese].
17. X. G. Cheng, Z. X. Li and Z. Y. Guo, *J. Eng. Thermophys.* **24**:94 (2003) [in Chinese].
18. K. L. Guo, *Numerical Heat Transfer* (AnHui Science and Technology House, Hefei, 1987), pp. 138–139.
19. Z. H. Xia, *Augmentation and Optimization on Heat Conduction and Convection Processes*, Ph.D. thesis (Tsinghua University, Beijing, 2001), pp. 25–32 [in Chinese].